Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

Mechanics M3

Advanced/Advanced Subsidiary

Wednesday 18 May 2016 – Morning

Time: 1 hour 30 minutes

Paper Reference **6679/01**

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. A particle P of mass 0.5 kg is moving along the positive x-axis under the action of a resultant force. The force acts along the x-axis. At time t seconds, P is x metres from the origin O and is moving away from O in the positive x direction with speed $\frac{12}{x+3}$ m s^{-1}.

(*a*) Find the magnitude of the force acting on P when $x = 3$.

(4)

Given that $x = 4$ when $t = 2$,

(*b*) find the value of t when $x = 10$.

(5)

(Total 9 marks)

2.

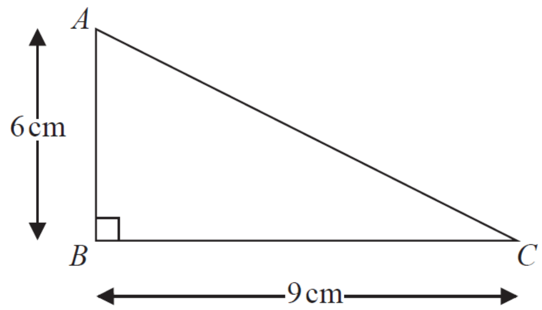

Figure 1

Figure 1 shows a uniform triangular lamina ABC in which AB = 6cm, BC = 9cm and angle ABC = 90°. The centre of mass of the lamina is G. Use algebraic integration to find the distance of G from AB.

(Total 6 marks)

---

3. One end of a light elastic string, of natural length 1.5m and modulus of elasticity 14.7 N, is attached to a fixed point O on a ceiling. A particle P of mass 0.6 kg is attached to the free end of the string. The particle is held at O and released from rest. The particle comes to instantaneous rest for the first time at the point A.

Find

(*a*) the distance OA,

(6)

(*b*) the magnitude of the instantaneous acceleration of P at A.

(3)

(Total 9 marks)

4.

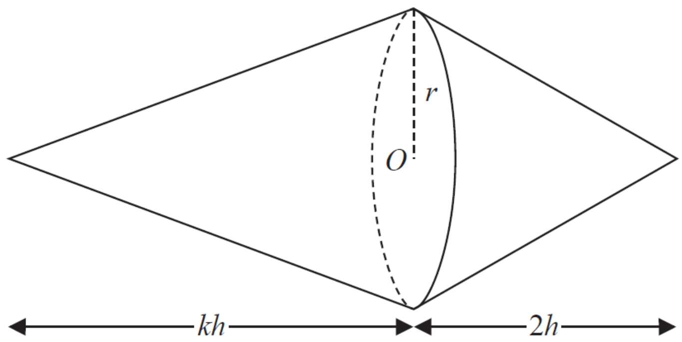

Figure 2

A uniform solid S consists of two right circular cones of base radius r. The smaller cone has height $2h$ and the centre of the plane face of this cone is O. The larger cone has height kh, where $k > 2$. The two cones are joined so that their plane faces coincide, as shown in Figure 2.

(*a*) Show that the distance of the centre of mass of S from O is

$$\frac{h}{4}(k-2).$$

(5)

The point A lies on the circumference of the base of one of the cones. The solid is suspended by a string attached at A and hangs freely in equilibrium.

Given that $r = 3h$ and $k = 6$,

(*b*) find the size of the angle between AO and the vertical.

(3)

(Total 8 marks)

5.

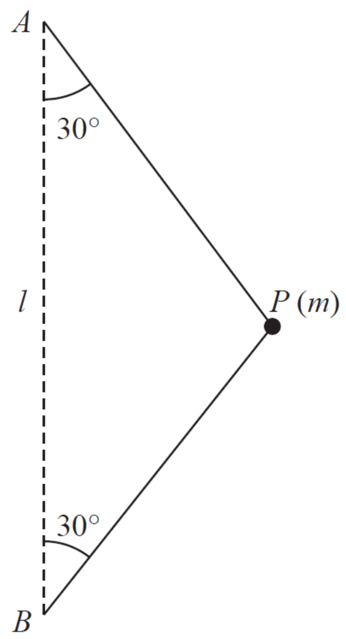

Figure 3

A particle P of mass m is attached to the ends of two light inextensible strings. The other ends of the strings are attached to fixed points A and B, where B is vertically below A and $AB = l$. The particle is moving with constant angular speed ω in a horizontal circle. Both strings are taut and inclined at 30° to AB, as shown in Figure 3.

(*a*) (i) Show that the tension in AP is $\frac{m\sqrt{3}}{6}(2g - l\omega^2)$.

(ii) Find the tension in BP.

(9)

(*b*) Show that the time taken by P to complete one revolution is less than $\pi\sqrt{\frac{2l}{g}}$.

(4)

(Total 13 marks)

6. One end of a light inextensible string of length l is attached to a particle P of mass $2m$. The other end of the string is attached to a fixed point A. The particle is hanging freely at rest with the string vertical. The particle is then projected horizontally with speed $\sqrt{\frac{7gl}{2}}$.

(*a*) Find the speed of P at the instant when the string is horizontal.

(4)

When the string is horizontal and P is moving upwards, the string comes into contact with a small smooth peg which is fixed at the point B, where AB is horizontal and $AB < l$. The particle then describes a complete semicircle with centre B.

(*b*) Show that $AB \geq \frac{1}{2}l$.

(9)

(Total 13 marks)

---

7. A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 1.2 m and modulus of elasticity 15 N. The other end of the spring is attached to a fixed point A on a smooth horizontal table. The particle is placed on the table at the point B where AB = 1.2 m. The particle is pulled away from B to the point C, where ABC is a straight line and BC = 0.8 m, and is then released from rest.

(*a*) (i) Show that P moves with simple harmonic motion with centre B.

(ii) Find the period of this motion.

(5)

(*b*) Find the speed of P when it reaches B.

(2)

The point D is the midpoint of AB.

(*c*) Find the time taken for P to move directly from C to D.

(3)

When P first comes to instantaneous rest a particle Q of mass 0.3 kg is placed at B. When P reaches B again, P strikes and adheres to Q to form a single particle R.

(*d*) Show that R also moves with simple harmonic motion.

(3)

(*e*) Find the amplitude of this motion.

(4)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

M3 6679 June 2016
Mark Scheme

Question Number	Scheme	Marks
1(a)	$v=\dfrac{12}{x+3}$	
	$\dfrac{\mathrm{d}v}{\mathrm{d}x}=-\dfrac{12}{(x+3)^2}$	M1
	$F=0.5v\dfrac{\mathrm{d}v}{\mathrm{d}x}=0.5\times\dfrac{12}{x+3}\times-\dfrac{12}{(x+3)^2}$	DM1A1
	$x=3\quad \lvert F\rvert=0.5\times\dfrac{12}{6}\times\dfrac{12}{6^2}=\dfrac{1}{3}\text{N}$	A1 (4)
(b)	$\int(x+3)\,\mathrm{d}x=\int 12\,\mathrm{d}t$	
	$\dfrac{1}{2}x^2+3x=12t\ (+c)$	M1A1
	$x=4,\ t=2\quad 8+12=24+c\qquad c=-4$	DM1
	$x=10\quad 50+30=12t-4$	DM1
	$t=7$	A1cao (5)
ALT (b)	Definite integration:	[9]
	$\int_2^T 12\,\mathrm{d}t=\int_4^{10}(x+3)\,\mathrm{d}x$	
	$12(T-2)=\left[\dfrac{x^2}{2}+3x\right]_4^{10}$ — M1A1 as main scheme - limits not needed	
	DM1 Correct limits shown	
	$12(T-2)=80-20,\quad T=7$ — DM1 Substitute limits, A1 $T=7$	

(a)M1 Attempt differentiation of $v=\dfrac{12}{x+3}$ or $\dfrac{1}{2}v^2=\dfrac{72}{(x+3)^2}$ wrt x $\quad(x+3)^{-2}$ or $(x+3)^{-3}$ (oe) must be seen. Both sides of the equation must be differentiated wrt x

DM1 Use NL2 with accel $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ as obtained above. Must include mass. Dependent on the first M mark.

A1 Correct expression for F with correct mass and correct acceleration seen here or before use in NL2

A1 Use $x = 3$ to obtain the correct magnitude, $\dfrac{1}{3}$, 0.33 or better **Must** be positive

(b)M1 Use $v=\dfrac{\mathrm{d}x}{\mathrm{d}t}$ and attempt the integration

A1 Correct integration constant of integration not needed

DM1 Use given values to obtain a value for c. Dependent on first M mark

DM1 Use $x=10$ to obtain a linear equation for t. Dependent on the first but **not** the second M mark

A1 cao $\quad t=7$

Question Number	Scheme	Marks
2	$\int xy\,dx = \int\left(-\frac{2x^2}{3}+6x\right)dx$	M1
	$=\left[-\frac{2}{9}x^3+3x^2\right]_0^9$	DM1A1
	$=-162+243-0=81$	A1
	$\bar{x}=\frac{81}{27}=3$	M1A1cso (6)
		[6]

M1 Attempting to obtain the correct form for $\int xy\,dx$, using an equation of a line. Limits not needed.

DM1 Attempting the integration. Limits not needed. Dependent on the first M mark.

A1 Correct integration and correct limits shown. This is **not** ft; equation of the line must be correct.

A1 Substitute the correct limits to obtain 81

M1 Divide their value from the integration by their area of the triangle

A1cso $\bar{x}=3$

ALTs:

1 Using *C* as the origin and *AB* parallel to *y*-axis:

Equation of line must be $y=\frac{2}{3}x$

$\int xy\,dx = \int\left(\frac{2x^2}{3}\right)dx == \left[\frac{2}{9}x^3\right]_0^9$ — M1DM1A1

$=162$ — A1

$\bar{x}=\frac{162}{27},(=6)$ Dist from $AB = 9-6=3$ — M1, A1

2 Using *AB* along the *x*-axis:

Must be using $\int \frac{1}{2}y^2 dx$

(i) Origin at *B* equation of line is $y=-\frac{3}{2}x+9$

(ii) Origin at *A* equation of line is $y=\frac{3}{2}x$

NB Ignore any work for the distance from *BC*, whether before or after distance from *AB*

Question Number	Scheme	Marks
3(a)	EPE gained $=\dfrac{14.7x^2}{2\times 1.5}$	B1
	$\dfrac{14.7x^2}{3}=0.6g\times(x+1.5)$	M1A1ft
	$5x^2-6x-9=0$	
	$x=\dfrac{6\pm\sqrt{36+180}}{10}=2.069...$ (or -0.869)	DM1A1
	$OA=3.569...=3.6$ or 3.57	A1 (6)
(b)	$\dfrac{14.7\times 2.069}{1.5}-0.6\times 9.8=0.6a$ or $0.6\times 9.8-\dfrac{14.7\times 2.069}{1.5}=0.6a$	M1A1ft
	$a=23.993...=$ 24 or 24.0	A1 cao (3) [9]

(a) B1 Correct EPE when extension is x

M1 Equating EPE to GPE lost EPE to be of the form $k\dfrac{\lambda x^2}{l}$, where k is a rational no.

A1ft Correct equation ft their EPE. Use of unknown must be consistent.

DM1 Solve their equation (3TQ) (quadratic formula must be correct)

A1 $x=2.069...$neg value not needed

A1 Add 1.5 to 2.069... and give final answer to 2 or 3 sf. (No "exact" answers allowed here due to use of g.)

ALT 1: Using extension $(x-1.5)$:

B1 Correct EPE when extension is $(x-1.5)$ where x is total length

M1 Equating EPE to GPE lost EPE of form shown above

A1ft Correct equation ft their EPE. Use of unknown must be consistent. No simplification needed. Equation is $\dfrac{14.7(x-1.5)^2}{3}=0.6gx$

A1 Simplify to $x^2-4.2x+2.25=0$ or equivalent 3TQ

DM1 Solve their equation (3TQ) (quadratic formula must be correct)

A1 $x=3.57$ or 3.6 Must be 2 or 3 sf

ALT 2: Use $v^2=u^2+2as$ or energy to obtain speed at natural length, then energy to A.

B1 for EPE at A

No more marks until an energy equation with EPE, GPE and KE terms seen.

M1A1ft Energy equation ft their EPE and initial speed EPE of form shown above

DM1A1
A1 As main scheme

NB: Solution of quadratic by calculator: Method mark only available if solution is correct (2.069 or 3.569)

(b)M1 Use NL2 at A inc use of Hooke's law. Formula for HL to be correct. Ext can be $(3.569-1.5)$

A1ft Correct numbers in the equation, ft their extension

A1 24 or 24.0 only. (No negatives allowed.)

Question Number	Scheme	Marks
4 (a)	Mass ratio $\frac{2}{3}\pi r^2 h$ $\quad \frac{1}{3}\pi r^2 kh$ $\quad \frac{2}{3}\pi r^2 h+\frac{1}{3}\pi r^2 kh$ (or 2 : k : (2 + k))	B1
	Dist from O $\quad -\frac{1}{2}h$ $\quad \frac{1}{4}kh$ $\quad \bar{x}$	B1
	$2\left(-\frac{1}{2}h\right)+k\times\frac{k}{4}h=(2+k)\bar{x}$	M1A1ft
	$\bar{x}=\frac{(k^2-4)h}{4(2+k)}=\frac{h(k-2)(k+2)}{4(2+k)}=\frac{h}{4}(k-2)$ *	A1cso (5)
(b)	$\tan\theta=\frac{\bar{x}}{r}$	M1
	$\tan\theta=\frac{32h}{4\times 8\times 3h}$	A1
	$\theta=18.43...$ or 0.321...rad Accept 18° or 0.32 rad or better	A1 (3) [8]

(a)B1 Correct **ratio** of volumes or masses - any form

B1 Correct distances from O or a vertex. One distance may be negative or all may be positive.

M1 Forming a moments equation. May be about O or either vertex.

A1ft Correct equation. All signs must be correct for their choice of point. Follow through the B marks.

A1cso Correct completion to the distance from O. (Factorisation **must** be shown.)

NB: First four marks available for $2\left(\frac{1}{2}h\right)-k\times\frac{k}{4}h=(2+k)\bar{x}$ **but** $k>2$ must be stated as a reason for changing $\frac{h}{4}(2-k)$ to the given answer.

(b)M1 Form an expression for $\tan\theta$ using the given $\bar{x}$ No need to substitute for r or k but $\bar{x}=h\Rightarrow$ correct $\bar{x}$ used. May be either way up

A1 Substitute for r and k to obtain a correct numerical (or equivalent to numerical) value for $\tan\theta$

A1 Correct angle, may be degrees or radians.

Question Number	Scheme	Marks
5(a)	$T_A\cos 30° = mg + T_B\cos 30°$	M1A1
	$T_A - T_B = \dfrac{2mg}{\sqrt{3}}$	
	Radius $= \dfrac{1}{2}l\tan 30° \quad \left(=\dfrac{\sqrt{3}}{6}l \text{ oe}\right)$	B1
	$T_A\cos 60° + T_B\cos 60° = mr\omega^2 = m\left(\dfrac{1}{2}l\tan 30°\right)\omega^2$	M1A1A1ft
	$T_A + T_B = \dfrac{ml\omega^2}{\sqrt{3}}$	
(i)	$T_A = \dfrac{1}{2}\left(\dfrac{2mg}{\sqrt{3}} + \dfrac{ml\omega^2}{\sqrt{3}}\right) = \dfrac{m\sqrt{3}}{6}\left(2g + l\omega^2\right)$ *	DM1A1cso
(ii)	$T_B = \dfrac{1}{2}\left(\dfrac{ml\omega^2}{\sqrt{3}} - \dfrac{2mg}{\sqrt{3}}\right)$ oe	A1cso (9)
(b)	$T_B > 0 \quad 2mg < ml\omega^2$	M1
	$\omega^2 > \dfrac{2g}{l}$	A1
	$T = \dfrac{2\pi}{\omega} \quad T < 2\pi\sqrt{\dfrac{l}{2g}} = \pi\sqrt{\dfrac{2l}{g}}$ *	DM1A1cso (4) [13]

(a)M1	Attempt a vertical equation, can have θ for the angle
A1	Completely correct equation, must have numerical angle now
B1	Correct radius seen anywhere
M1	NL2 along the radius. Acceleration in either form and can have r for the radius
A1	Correct sum of tensions (may have a tension on each side)
A1ft	Correct mass x acceleration, follow through their radius
(i)DM1	Solve the equations to either T_A =... or T_B = ... Dependent on both previous M marks. Can be awarded for finding T_A or T_B
A1cso	Correct expression for T_A Given answer so no equivalents allowed.
(ii)A1 cso	Correct expression for T_B. Any equivalent 2 term expression allowed.
Special case	If only one of vertical and radial equations found and the given T_A used to find T_B, award the marks earned for the equation and radius, if used, and B1 for T_B (last A1 in (a) on e-PEN) Max score 5/9
(b)M1	Deducing an inequality from the expression for T_B Can have $2(m)g < (m)l\omega^2$ or $2(m)g \leqslant (m)l\omega^2$ or $2(m)g = (m)l\omega^2$
A1	$\omega^2 > \dfrac{2g}{l}$ or $\omega^2 \geqslant \dfrac{2g}{l}$ oe inc equivalent in words.
DM1	Use $T = \dfrac{2\pi}{\omega}$ with their ω to form an inequality for T, can have $T < ...$ or $T \leqslant ...$ Dependent on the first M mark of (b)
A1cso	For a correct final statement from a correct solution. Must be $T < \ldots$ or equivalent in words

Question Number	Scheme	Marks
6	Energy to horizontal: $\frac{1}{2}\times 2m\times\frac{7gl}{2}-\frac{1}{2}\times 2mv^2=2mgl$	M1A1A1
	$v=\sqrt{\frac{3gl}{2}}$	A1 (4)
(b)	Energy from horizontal to top: $\frac{1}{2}\times 2m\times\frac{3gl}{2}-\frac{1}{2}\times 2mV^2=2mgr$	M1A1
	$V^2=\frac{3gl}{2}-2gr$	
	NL2 at top: $\frac{2mV^2}{r}=2mg+T$	M1A1A1
	$T\geqslant 0\Rightarrow \frac{2mV^2}{r}\geqslant 2mg$	M1
	$\frac{3gl-4gr}{2r}\geqslant g$	DM1
	$\frac{3gl}{2}-2gr\geqslant gr$	
	$r\leqslant\frac{1}{2}l$	A1
	$AB\geqslant\frac{1}{2}l$ *	A1cso (9) [13]

(a)M1 Attempting an energy equation to the horizontal. Must be clear energy is being used and not $v^2=u^2+2as$. Mass can be *m* or 2*m*. Mixed masses are accuracy errors.

A1 Correct difference of KE terms Mass can be *m* or 2*m*

A1 Correct PE and all signs correct in equation. Mass can be *m* or 2*m* but mixed masses score A0.

A1 Correct speed at the horizontal (regardless of mass used).

(b)M1 Attempt energy equation from the horizontal to the top of the new (smaller) circle with unknown radius OR from lowest point of original circle to top of the new circle. Mass can be *m* or 2*m* or mixed.

A1 Correct equation. Mass can be *m* or 2*m* (but same in all terms).

M1 NL2 at top of the small circle. Mass can be *m* or 2*m* or mixed. Allow with $T=0$

A1 Correct mass x acceleration. Mass can be *m* or 2*m*

A1 Both force terms correct. Mass can be *m* or 2*m* but must be the same as used in the acceleration term.

M1 Use $T\geqslant 0$ to obtain an inequality for V^2. Allow if *T* assumed to be zero in NL2.

DM1 Use the energy equation to eliminate V^2 Dependent on first and second M marks. These two method steps may occur in the reverse of the order shown here.

A1 Correct maximum value for *r*

A1cso Correct inequality for *AB*. **This is cso. Candidates who have used *m* in NL2 or assumed *T* = 0 cannot be awarded this mark.**

ALT: Combining lines 3 and 4 of (b):

$\frac{2mV^2}{r}\geqslant 2mg$ scores M1A1A1M1 (All other marks as above.)

NB Equations to/at general positions do not gain marks until correct size of angle used.

Question Number	Scheme	Marks
7(a)	$T = -m\ddot{x}$	M1
	$\frac{15x}{1.2} = -0.5\ddot{x}$	M1A1
(i)	$\ddot{x} = -25x \quad \therefore$ SHM	A1cso
(ii)	Period = $\frac{2\pi}{5}$ (= 1.256 Accept 1.3 or better)	B1ft (5)
(b)	$v^2 = \omega^2\left(a^2 - x^2\right)$	
	$x = 0 \quad v = 5 \times 0.8 \quad v = 4 \text{ m s}^{-1}$	M1A1 (2)
(c)	$x = a\cos\omega t$	
	$x = -0.6 \quad -0.6 = 0.8\cos 5t$	M1A1ft
	$t = \frac{1}{5}\cos^{-1}\left(-\frac{6}{8}\right) = 0.4837...\text{s}$ accept 0.48 or better	A1cso (3)
(d)	$T = -m\ddot{y}$	
	$\frac{15y}{1.2} = -0.8\ddot{y}$	M1A1
	$\ddot{y} = -15.625y$ or $\ddot{y} = -\frac{125}{8}y \quad \therefore$ SHM	A1 (3)
(e)	Con of mom: $0.5 \times 4 = (0.5 + 0.3)V$	M1
	$V = \frac{2}{0.8} = 2.5 \text{ m s}^{-1}$	A1
	$(2.5)^2 = 15.625a^2$	DM1
	$a^2 = \frac{2.5^2}{15.625}$	
	$a = 0.6324....\text{m}$ accept 0.63 or better or $a = \frac{\sqrt{10}}{5}$ oe	A1 cso (4)
		[17]

Question Number	Scheme	Marks

(a)M1	Using NL2 with T for tension, acceleration a or $\ddot{x}$
M1	Using HL to obtain an equation connecting $\ddot{x}$ or a and x
A1	A correct equation - any equivalent to that shown - **must** have $\ddot{x}$ now.
(i)A1	$\ddot{x} = -25x$ **and** stating SHM
	These 4 marks are available without substituting for any or all of m, λ or l
(ii)B1ft	Period = $\dfrac{2\pi}{\text{their numerical } \omega}$ or decimal equivalent
(b)M1	Using $v^2 = \omega^2\left(a^2 - x^2\right)$ with $x = 0$ (or just $v = a\omega$) and $a = 0.8$, their ω
	OR: using $v = -a\omega \sin \omega t$ with $t = \dfrac{1}{4} \times \text{their period}$, $a = 0.8$, their ω
A1	Correct value for v
ALT:	Using energy: $\dfrac{1}{2} \times 0.5v^2 = \dfrac{15 \times 0.8^2}{2 \times 1.2}$ M1 $v = 4$ A1
(c)M1	Using $x = a\cos \omega t$ with their ω and $a = 0.8$, $x = \pm 0.6$ (or any other complete method)
	Use of $x = a\sin \omega t$ requires further work to complete the method.
A1ft	Correct equation follow through their ω
A1cso	$t = 0.48$ or better.
(d)M1	Using NL2 with tension at the new extension **and increased** mass (any variable inc x for extension), acceleration in differential form or just a
A1	Correct equation, any equivalent form, acceleration in differential form or a
A1	$\ddot{y} = -15.625y$ **and** stating SHM (unless already penalised in (a)) **must** have differential form for acceleration
(e)M1	Using the conservation of momentum equation with their speed of P at B (see ans to (b)
A1	Correct speed for R at B (no ft)
	Momentum equation often seen in (d). Marks can be awarded if the result is seen in (e)
DM1	Using $v^2 = \omega^2\left(a^2 - y^2\right)$ with $y = 0$ (or just $v = a\omega$) with their speed for R and their ω
	Dependent on the first M mark of (e)
A1cso	$a = 0.63$ or better or exact answer